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Data-dependent and data-independent methods of Maximum Time Interval Error assessments

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***Abstract*-In the paper the data-dependent and data-independent methods of the Maximum Time Interval Error assessment are presented and compared. The pros and cons of the methods are considered. The solutions of time effective MTIE assessment for several cases are suggested.**

I. INTRODUCTION

The Maximum Time Interval Error (MTIE) is the important characteristic of the timing signals in the telecommunication network. MTIE is calculated using a series of equally spaced time error samples measured at some network interface. The number of data and the complexity of the estimator's formula cause that the time of the parameter's calculation using direct method (plain computation) is rather long. Rather short time spent on the estimation process can simplify the evaluation of the analyzed timing signal.

Several methods enabling the shortening the computation time were proposed in the literature. The paper is the comparison study of the methods described. Some of the methods [4, 5] based on the statistical characteristic are not accurate and give the approximate value of the parameter. The methods considered in the paper enable obtaining the accurate value of the parameter characterizing analyzed data series. Some of the methods proposed by the authors of the paper are data-dependent, because the random behavior of the data is used to speed up the computation process [6, 7, 8]. Another method (data-independent), proposed by Bregni and Maccabruni [9], uses the binary decomposition of the time error sequence for the data reducing and making faster the parameter's computation.

II. MAXIMUM TIME INTERVAL ERROR

The maximum time interval error is defined in international standards as the maximum peak-to-peak time error variation of a given timing signal, with respect to an ideal timing signal within a particular time period [1, 2, 3]. If the results of time error function measurements $x(t)$ take the form of N equally spaced samples $\{x_i\}$, MTIE can be estimated from the formula

$$M\hat{TIE}(n\tau_0) = \max_{1\le k\le N-n}\left(\max_{k\le i\le k+n} x_i - \min_{k\le i\le k+n} x_i\right) \quad (1)$$

where $\{x_i\}$ is a sequence of N samples of time error function $x(t)$ taken with sampling interval τ_0, $\tau=n\tau_0$ is an observation interval, and n=1, 2, ..., N–1.

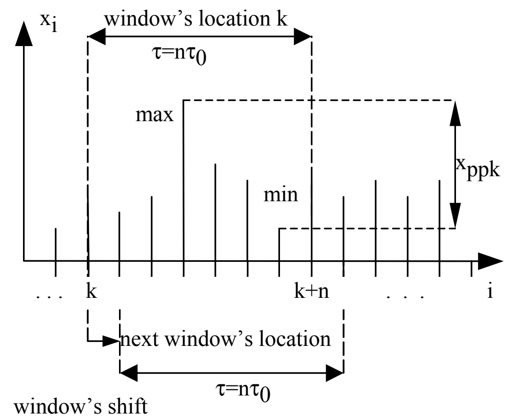

Fig. 1. The idea of direct search for MTIE

Following the formula (1) directly in order to find the estimate of MTIE for the observation interval τ, all intervals having the width of τ, existing in the sequence of N time error samples must be reviewed. The window having the width of $\tau=n\tau_0$ and including n+1 samples is set at the beginning of the data sequence $\{x_i\}$ and then it is shifted with the step of τ_0 to the end of the sequence. For each window's location the peak-to-peak value of time error in the window is found. The maximum peak-to-peak value found for all existing locations of the window is the value of MTIE(τ) estimate. The process of window reviewing does not depend on the data value. The complexity of calculation grows with n and therefore the direct method is really time-consuming. The example of window's shifts using the direct method for the 6-sample observation interval and time error sequence containing 32 samples is presented in Fig. 2.

III. DATA-DEPENDENT METHODS OF MTIE ASSESSMENT

The reviewing of each window's location during MTIE search using the direct method described above cause, that the calculation process is time consuming. The process of the MTIE search can be shorter, if some window's locations are excluded from inspection. Two methods, which were proposed [6] by the authors of this paper, enable such excluding without consequence for the accuracy of the calculation result.

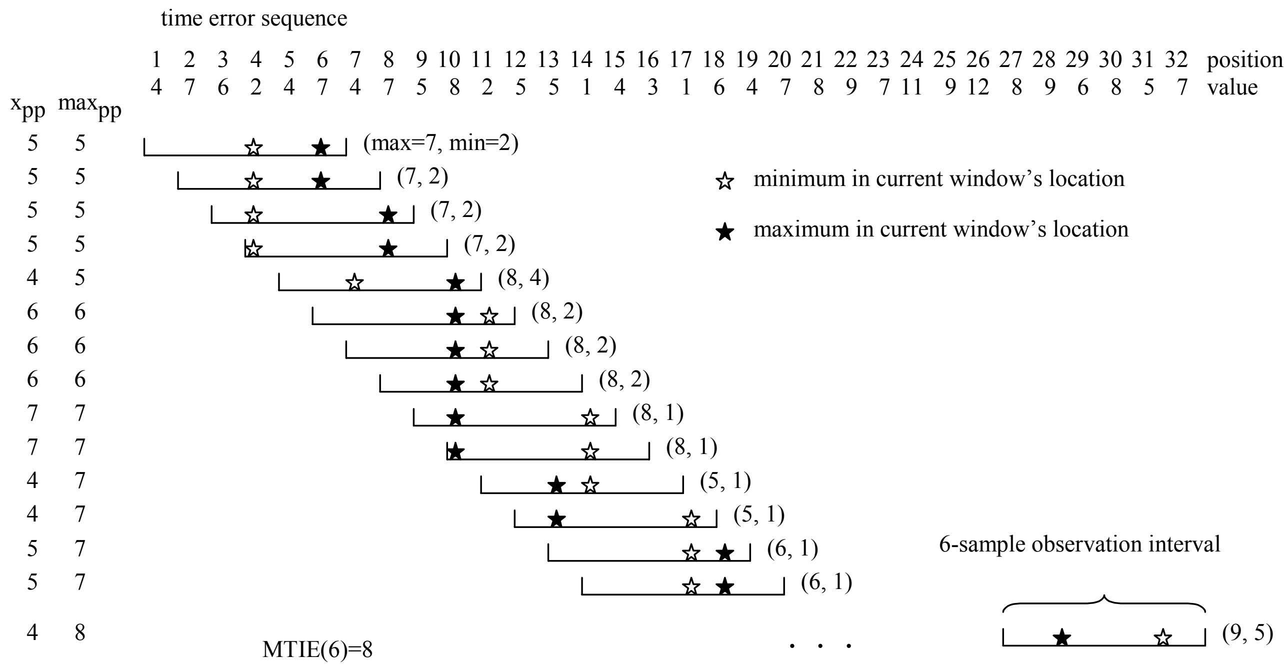

Fig. 2. Example of window's shifts using direct method for 6-sample observation interval

A. Boundaries decision method

In the process of the MTIE search using the boundaries decision (BD) method the window is shifted with the step of τ_0, but the decision on whole window's review depends on the values of the samples at the window's boundaries. Two samples: the earliest value, which leaves the window and the new sample, which appears at the window end (Fig. 3) are compared with current maximum and minimum samples. The result of the comparison determines the next operation. The new extreme value should be searched, when the current extreme sample leaves out the window and simultaneously the new value is not the new extreme.

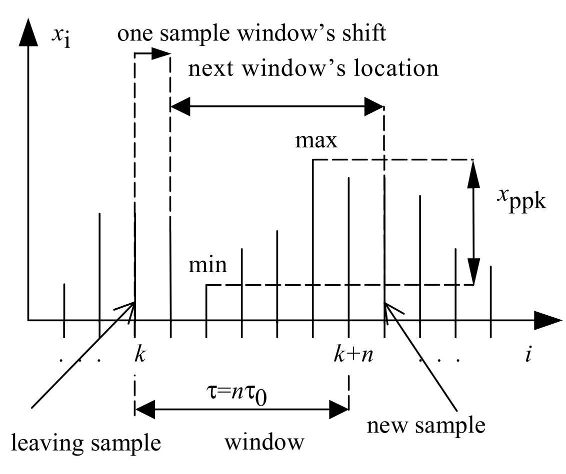

Fig. 3. The window's shift in the boundary decision method

B. Extreme fix method

In the process of the MTIE search using extreme fix (EF) method some window's locations are excluded from inspection if the peak-to-peak value for each of these locations is not greater than the value found until now, or if a greater peak-to-peak value may be found for the next window's locations. The EF method is based on fixing the positions of minimum and maximum samples for a given window's location. After finding the positions of the extremes the window's shift to the position of the first extreme (denoted as p_1) is performed (Fig. 4). Within the interval between the starting position of the previous window's location and the p_1 position there are no "more extreme" values than the ones which has been already found. After the shift the peak-to-peak value for the window's location p_1 should be found. Because the samples between the position p_1 and the last sample in the previous window's location (k+n) were reviewed and the extreme values are known, they are excluded from inspection. The one-sample window's shift is performed when the first sample in the window is the extreme sample. What will be done next depends on the values at the boundaries of the window, as in the BD method. The example of the MTIE search for the time error sequence containing 32 samples using EF method is presented in Fig. 5. It is worthwhile to mention, that using the EF method we must operate on the additional variables that carry the positions of the extreme samples. It was not necessary for the plain computation as well as boundaries decision method. The operations on the additional variables enable the considerable window's shifts, but in some cases (e.g. for small observation intervals) may complicate the MTIE search process.

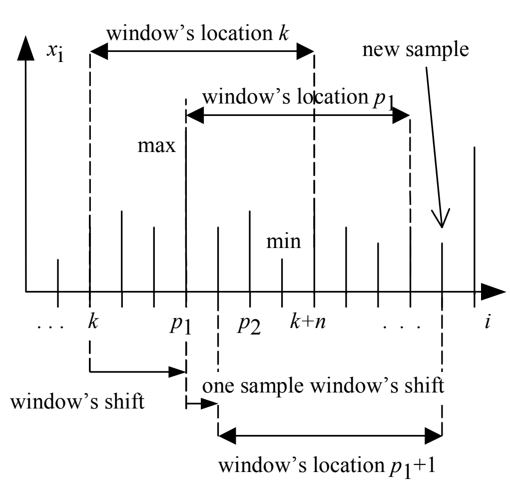

Fig. 4. The window's shift in the extreme fix method

C. Extreme fix method with sequential data reducing

Some features of the EF method are used for developing the method of MTIE calculation with sequential data reduction [7, 8]. This method is destinated for the MTIE assessment for a series of observation intervals starting from some τ_{min} until some τ_{max}. The basis of the method is a fact, that during the MTIE search process for some observation interval τ_i ($\tau_i > \tau_{min}$) we find the extreme samples for some window's location from the set of extreme samples found previously during the MTIE search for the smaller observation interval τ_{i-1} ($\tau_i > \tau_{i-1}$). Only these samples may influence the $M\hat{T}IE$ value for the observation interval τ_i. Other time error samples in the time error sequence do not matter in the MTIE search process. Therefore we can reduce the number of time error samples used for the MTIE calculation.

In general the procedure of the MTIE calculation for the row of the observation intervals from τ_{min} till τ_{max} will be as follows:

1. MTIE estimate calculation using EF method for the observation interval τ_{min} and data reduction;
2. MTIE estimate calculation using EF method for the observation intervals longer than τ_{min} using the reduced data being the series of extremes previously found.

The procedure was called the extreme fix method with sequential data reduction (EFSDR) [8]. The example of the MTIE search using the EFSDR method for the reduced data taken from the example in Fig. 5 is presented in Fig. 6.

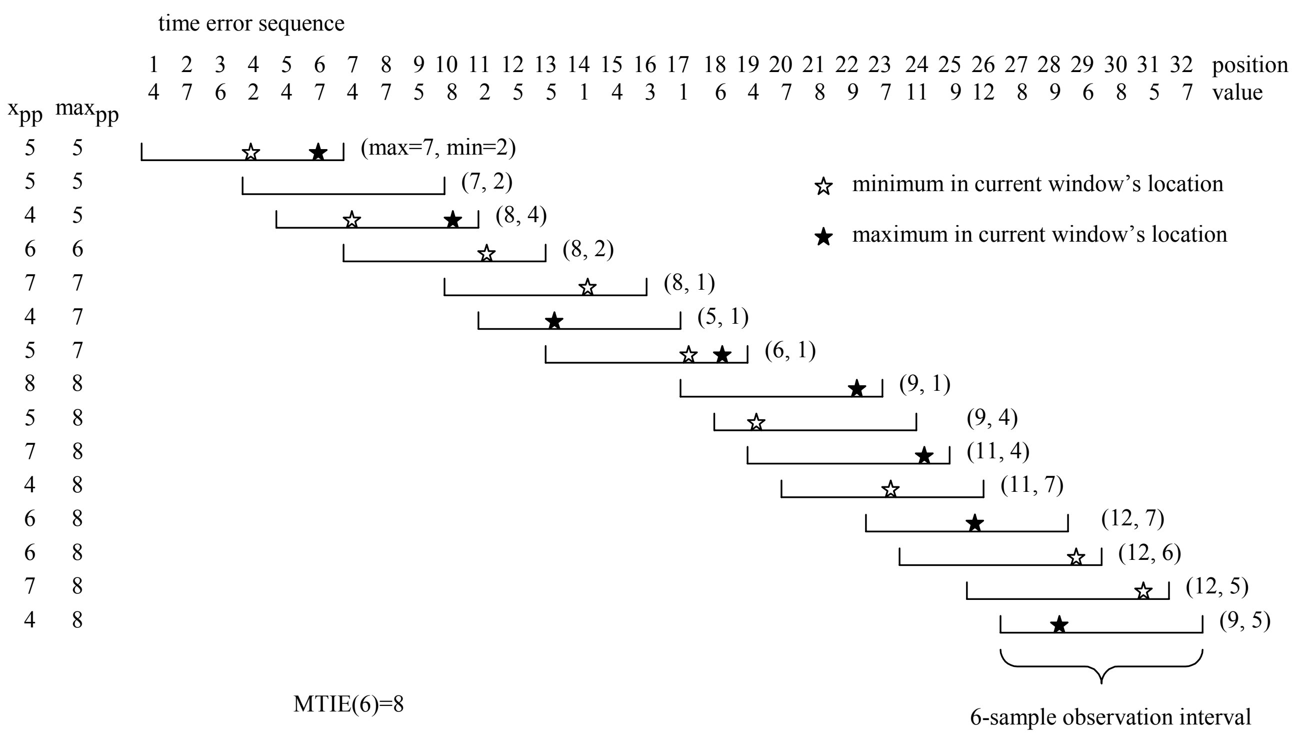

Fig. 5. Example of window's shifts using EF method for 6-sample observation interval

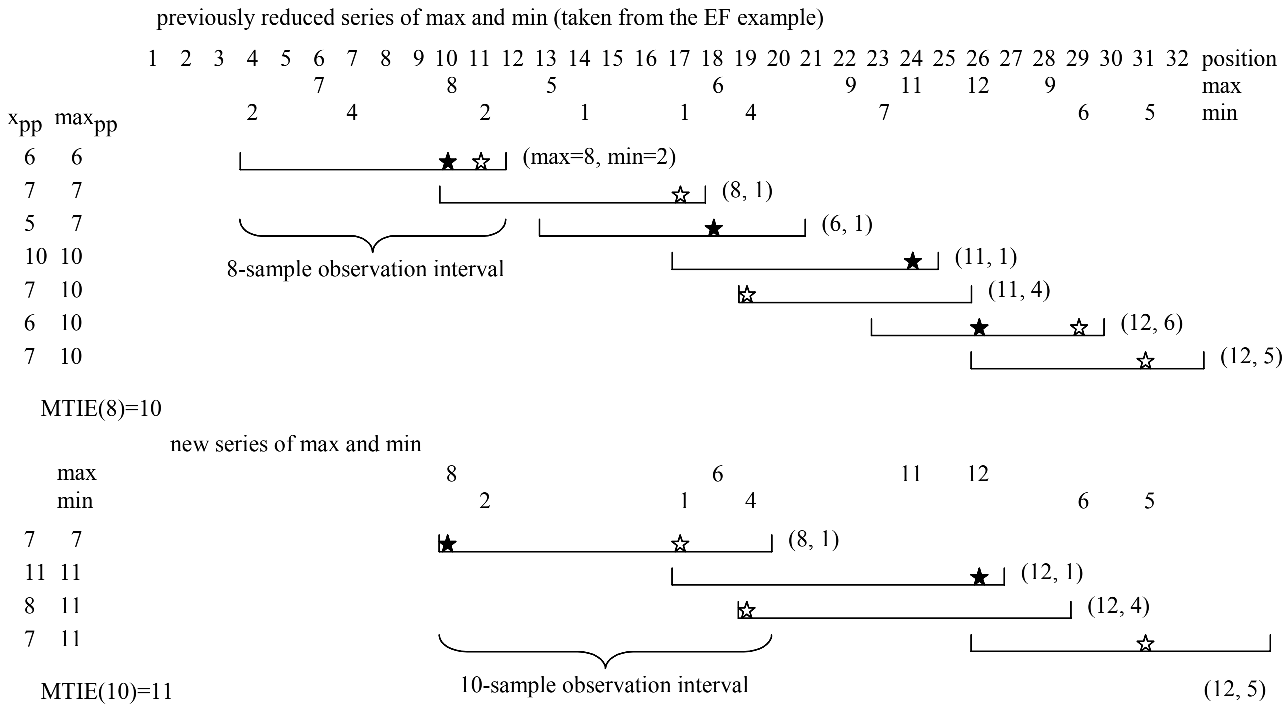

Fig. 6. Example of window's shifts using EFSDR method for 8-sample and 10-sample observation intervals

IV. MTIE Assessment With Binary Decomposition

The method of MTIE calculation proposed in [9] is based on the binary decomposition of the sequence containing time error samples which number must be equal to a power of 2. This method is destinated for the MTIE assessment for a series of observation intervals, as the EFSDR method is. Two main differences are: the data-independence of the data reduction, and the specific values of the observation intervals considered – the lengths of the intervals given by the number of samples must be powers of 2.

At the first step of the calculation process the 2-samples windows are considered. Two neighboring samples are compared and the maximum and minimum values for each pair are selected. The maximum of the difference between them within one window is the MTIE for the 2-samples observation interval.

At the second step the 4-samples windows are considered. All of the neighboring 2-samples windows are joined, and the previously selected values are used for finding the maximum and minimum within so created 4-samples windows (the comparison of the extremes in the neighboring pairs is performed only). Then the MTIE is searched.

At the successive steps the next windows (containing increasing number of samples) are considered by the creation from the previously analyzed windows, and the data reduction process proceeds. The example of the MTIE computation using binary decomposition for the series of 32 time error samples is presented in Fig. 7.

V. Comparison of The Methods

The MTIE is assessed usually for a series of observation intervals starting from some τ_{min} until some τ_{max}. It is performed especially when we want to check out whether or not the timing signal possesses the recommended quality [1, 2, 3]. Each of the method described in the previous sections can be used for such calculation, but the EFSDR method and the method with binary decomposition (denoted as BIN) were especially derived for this goal. Therefore these two methods will be compared in this section.

The first difference between these methods is the length of time error sequence used for MTIE calculation. The EFSDR method can be used for MTIE calculation for any length of the data sequence greater than the maximum observation interval τ_{max}. In the case of BIN method the length of the data sequence given by the number of time error samples must be equal to a power of 2. The consequence of this restriction is that the short cut of the sequence to the requested length must be performed, if we want to use the BIN method. The remaining part of the sequence is not used in the calculation and do not affect the parameter's value obtained.

Next difference is the length of the observation intervals for which the MTIE calculation may be performed. Using the EFSDR method the calculation can be done for any observation interval. Using the BIN method the value of MTIE can be calculated only for observation intervals having the number of samples equal to a power of 2. Therefore we can calculate the MTIE for any increasing series of observation intervals (e.g. containing 5, 50, 500 samples) using EFSDR method. The calculation using BIN method can be performed only for the series of observation intervals, which lengths given by the number of samples create the power of 2 series.

One of the main differences between these methods is the way of the data reduction. The process of the data reduction using the EFSDR method runs irregularly because the window's shifts over the data sequence are dependent on the data type. The data reduction process in the case of BIN method runs regularly and it is independent on the data type. Because of the regular data reduction there is no extreme cases from the point of view of data behavior when the BIN method is used for the MTIE calculation. The calculation process runs equally well for each data type. The extreme cases may happen when the EFSDR method is used. The calculation runs very fast for random data behavior (like white phase noise) when the considerable window's shifts are performed. When the monotonic change of data dominates the short-term random behavior, the calculation process runs rather slow. In such a case the extreme samples are located on the opposite boundaries of the window. Thus one-sample shift and window review procedure must be performed for the majority of window's locations [6, 8]. This situation occurs also when the BD and EF methods are used [6].

VI. Examples

In order to compare the MTIE calculation process using the EFSDR and BIN methods we consider the following examples.

A. Calculation for single observation interval

We have a sequence of 1024 time error samples and we need to compute the MTIE value for the observation interval containing 32 samples. It is very simple task using BD or EF methods, which are useful for calculation for single observation interval. The plain computation using direct method for these specific (rather short) lengths can be considerable short in time. The computation process using EFSDR method is performed in two steps. First the calculation and data reduction using EF method for any shorter observation interval must be done. Then using reduced data the final calculation for the requested 32 samples interval is performed. When the BIN method is used, the calculation for each preceding interval (having power of 2 samples and smallest then 32, i.e. 2, 4, 8, and 16) must be performed first. Finally the calculation for the 32 samples interval using previously reduced samples is done. Because of the initial steps the calculation using binary decomposition is more complicated for that case than using other methods.

If the number of samples in the required observation interval is not equal to power of 2 (e.g. 40), then we may obtain using BIN method only rough estimation of the real MTIE value. In this case we must choose the value of power of 2 closer to the required length of observation interval, and make the computation for this chosen length. Therefore the method with binary decomposition seems to be not suitable for MTIE calculation for single observation interval.

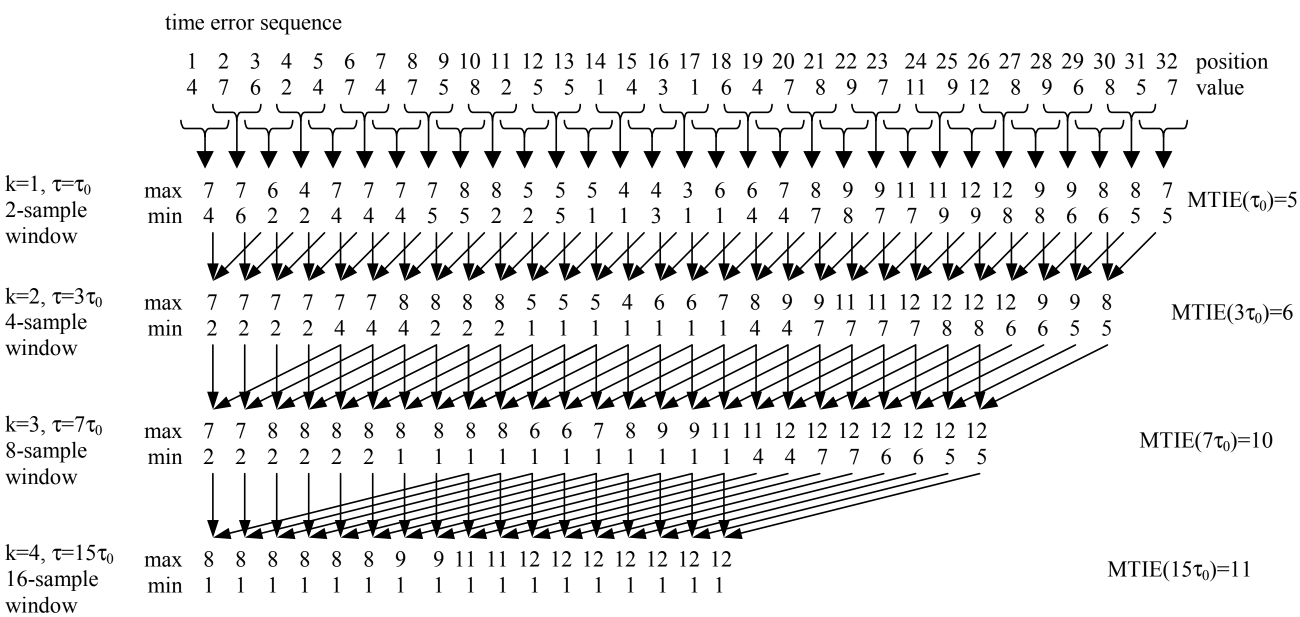

Fig. 7. Example of the MTIE search using binary decomposition

B. Calculation for series of observation intervals

We have a time error sequence taken with sampling interval τ_0=0.1 s during the measurement time T=4000 s (40 001 samples) and we need to compute the MTIE value for the series of observation intervals: 1 s, 10 s, 100 s and 1000 s. Computation by means of the EFSDR method consists of four following steps:

1. step: computation of MTIE for the observation interval τ_{min}=1 s (11 samples window) using EF method together with data (max and min) selection;
2. step: computation of MTIE for the observation interval τ=10 s (101 samples window) using previously reduced data together with data selection;
3. step: computation of MTIE for the observation interval τ=100 s (1001 samples window) using previously reduced data together with data selection;
4. step: computation of MTIE for the observation interval τ=1000 s (10001 samples window) using previously reduced data.

As result we have four MTIE values for the required observation intervals.

Computation by means of the BIN method consists of the following steps:

an initial step: short cut of the TE sequence to the length of 32 768 samples

1. step: computation for 2-sample observation interval and data reduction;
2. step: computation for 4-sample observation interval and data reduction;
3. step: computation for 8-sample observation interval and data reduction;

4-13: successive steps for windows having 16, 32, … , 8192 samples and data reduction;

14. step: computation for 16384-sample observation interval.

As result we have a series of fourteen MTIE values and we must approximate the lengths of observation intervals (8 or 16 instead of 11 samples, 64 or 128 instead of 101, 512 or 1024 instead of 1001, 8192 or 16384 instead of 10001). In addition the remaining part of the sequence was not used in computation process and it does not affect the obtained result.

VII. Conclusions

The data-dependent methods of MTIE assessment are flexible in the terms of the lengths of observation intervals and data sequence. The calculation using BD, EF and EFSDR method can be done for any length of data sequence and any lengths of observation intervals. The extreme fix method and boundaries decision method are suitable for MTIE assessment for a single observation interval. Therefore we can express the following suggestions.

The data-dependent EFSDR method of MTIE calculation is suggested for the parameter assessment for the time error sequences showing short-term random behavior and when we have rigorously defined observation intervals.

The data-independent method of MTIE calculation using binary decomposition is to be applied for the monotonic changes of time error data, when we have no rigorously given values of the observation intervals and the length of the data series.

Time error sequences taken from the measurement in the telecommunications networks (where MTIE is a valuable parameter) contain sufficient random component, which makes the EFSDR method very effective. If the type of the data sequence is unknown, the preceding calculation of Allan deviation or autocorrelation function can be helpful for selection of the effective MTIE assessment method.

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